

University of Groningen

Classification of supersymmetric backgrounds of string theory

Gran, Ulf; Gutowski, Jan; Papadopoulos, George; Roest, Diederik

Published in:
Fortschritte der Physik

DOI:
[10.1002/prop.200610363](https://doi.org/10.1002/prop.200610363)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2007

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Gran, U., Gutowski, J., Papadopoulos, G., & Roest, D. (2007). Classification of supersymmetric backgrounds of string theory. *Fortschritte der Physik*, 55(5), 736-741.
<https://doi.org/10.1002/prop.200610363>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Classification of supersymmetric backgrounds of string theory

Ulf Gran¹, Jan Gutowski², George Papadopoulos³, and Diederik Roest^{4*}

¹ Institute for Theoretical Physics, K. U. Leuven, Celestijnenlaan 200D, 3001 Leuven, Belgium

² DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, UK

³ Department of Mathematics, King's College London, Strand, London WC2R 2LS, UK

⁴ Departament Estructura i Constituents de la Matèria, Facultat de Física, Universitat de Barcelona, Diagonal, 647, 08028 Barcelona, Spain

Received 1 December 2006, accepted 5 February 2007

Published online 15 May 2007

Key words Supergravity, supersymmetry, string/M-theory

PACS 11.25.-w, 11.25.Yb

We review the recent progress made towards the classification of supersymmetric solutions in ten and eleven dimensions with emphasis on those of IIB supergravity. In particular, the spinorial geometry method is outlined and adapted to nearly maximally supersymmetric backgrounds. We then demonstrate its effectiveness by classifying the maximally supersymmetric IIB G -backgrounds and by showing that $N = 31$ IIB solutions do not exist.

© 2007 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim

1 Introduction

The supersymmetric solutions of $D = 10$ and $D = 11$ supergravities are instrumental in the understanding of string/M-theory dualities, compactifications and the AdS/CFT correspondence. Most of these solutions have been found using Ansätze adapted to the requirements of physical problems. However, the realization that there are new maximally supersymmetric solutions [1], the rediscovery of some old ones [2, 3], and their subsequent applications in AdS/CFT correspondence, have indicated that a more systematic investigation of supersymmetric solutions in supergravity theories is needed. By solving $\mathcal{R} = 0$, where \mathcal{R} is the supercovariant curvature, the authors of [4] classified the maximally supersymmetric solutions of $D = 10$ and $D = 11$ supergravities. The G -structure method, based on the Killing spinor form bi-linears and refined in [5], has also been used in [6, 7] to solve the Killing spinor equations (KSE) of $N = 1$ backgrounds of $D = 11$ supergravity, i. e. the backgrounds that admit one Killing spinor.

The spinorial geometry method of solving Killing spinor equations, proposed in [8], is based on the gauge symmetry of Killing spinor equations, on a description of spinors in terms of forms, and on an oscillator basis in the space of spinors. In $D = 11$, it has been applied to considerably simplify the solution of the KSE for $N = 1$ backgrounds, and then to investigate backgrounds with two, three, four and 31 supersymmetries. Furthermore, spinorial geometry has been used to solve the KSE of IIB $N = 1$ backgrounds [9, 10], and to explore the geometry of supersymmetric heterotic backgrounds [11]. In this talk two of the most recent applications in IIB are reviewed. These are the supersymmetric backgrounds with the maximal number of G -invariant Killing spinors [10, 12], and $N = 31$ backgrounds [13].

* Corresponding author E-mail: droest@ecm.ub.es, Phone: +00 34 934 037 062, Fax: +00 34 934 021 198

2 IIB maximally supersymmetric G -backgrounds

Supersymmetric backgrounds can be characterized by the number N of Killing spinors and their stability subgroup G in an appropriate spin group [14]. For a given stability subgroup G , the KSE of IIB supergravity simplify for backgrounds that admit the maximal number of G -invariant Killing spinors [10, 15]. Such backgrounds can be thought of as the vacua of IIB strings in a compactification scenario. In particular, it has been found that the Killing spinors can be written as

$$\epsilon_i = \sum_{j=1}^N f_{ij} \eta_j, \quad j = 1, \dots, N = 2m, \quad (1)$$

where η_p , $p = 1, \dots, m$ is a basis of G -invariant Majorana-Weyl spinors, $\eta_{m+p} = i\eta_p$, and (f_{ij}) is an $N \times N$ invertible matrix of real spacetime functions. It turns out that in such cases the IIB KSE and their integrability conditions factorize [10, 15].

IIB Killing spinors are invariant under the subgroups $Spin(7) \ltimes \mathbb{R}^8$ (2), $SU(4) \ltimes \mathbb{R}^8$ (4), $Sp(2) \ltimes \mathbb{R}^8$ (6), $(SU(2) \times SU(2)) \ltimes \mathbb{R}^8$ (8), \mathbb{R}^8 (16), G_2 (4), $SU(3)$ (8), $SU(2)$ (16) and $\{1\}$ (32) of $Spin(9, 1)$, where the number in parenthesis denotes the maximal number of invariant spinors in each case. These groups have been found in the context of the heterotic string [11]. The $\{1\}$ ($N = 32$) case consists of the maximally supersymmetric backgrounds which have been classified in [4]. These are locally isometric to Minkowski spacetime $\mathbb{R}^{9,1}$, $AdS_5 \times S^5$ [16] and the maximally supersymmetric Hpp-wave [1]. The remaining cases have been classified in [10, 12]. It is instructive to distinguish between compact and non-compact stability subgroups in $Spin(9, 1)$ because the geometry is different in these two cases.

First consider the supersymmetric backgrounds associated with the compact stability subgroups $G = G_2, SU(3)$ and $SU(2)$. The spacetime M of such backgrounds is locally isometric to a product $M = X_n \times Y_{10-n}$ with $n = 3, 4, 6$, where X_n is a maximally supersymmetric solution of an n -dimensional supergravity theory and Y_{10-n} is a Riemannian manifold with holonomy G . In the G_2 case $X_3 = \mathbb{R}^{2,1}$ and Y_7 is a holonomy G_2 manifold. In the $SU(3)$ case, $X_4 = AdS_2 \times S^2$, $\mathbb{R}^{3,1}$ or CW_4 and Y_6 is a Calabi-Yau manifold, where CW_4 is a 4-dimensional Cahen-Wallach plane wave. Similarly in the $SU(4)$ case, $X_6 = AdS_3 \times S^3$, $\mathbb{R}^{5,1}$ or CW_6 and Y_4 is hyper-Kähler. Apart from the cases in which X_n is flat, all these backgrounds have non-trivial fluxes and the full solutions can be found in [12].

Next we summarize the geometry and fluxes of supersymmetric backgrounds associated with non-compact stability subgroups $G = K \ltimes \mathbb{R}^8$ for $K = Spin(7), SU(4), Sp(2), SU(2) \times SU(2)$ and $\{1\}$, for a detailed exposition see [12]. In all these cases, the spacetime M admits a null parallel vector field X and the holonomy of the Levi-Civita connection, ∇ , of spacetime is contained in $K \ltimes \mathbb{R}^8$, i.e.

$$\nabla_A X = 0, \quad \text{hol}(\nabla) \subseteq K \ltimes \mathbb{R}^8. \quad (2)$$

Therefore, the spacetime is a pp-wave propagating in an eight-dimensional Riemannian manifold Y_8 such that $\text{hol}(\tilde{\nabla}) \subseteq K$, where $\tilde{\nabla}$ is the Levi-Civita connection of Y_8 . Alternatively, the spacetime is a two-parameter Lorentzian deformation family of Y_8 . Adapting coordinates along the parallel vector field $X = \partial/\partial u$, the metric can be written as

$$ds^2 = 2dv(du + Vdv + n) + ds^2(Y_8), \quad ds^2(Y_8) = \gamma_{IJ} dy^I dy^J = \delta_{ij} e^i_I e^j_J dy^I dy^J \quad (3)$$

where V , n , and the metric $ds^2(Y_8)$ may also depend on the coordinate v . In all cases, the fluxes are null,

$$P = P_-(v)e^-, \quad G = e^- \wedge L, \quad F = e^- \wedge M, \quad (4)$$

and the Bianchi identities give $dP = dG = dF = 0$, where L and M are a two- and a self-dual four-form, respectively, of Y_8 . In particular, one finds that $P_- = P_-(v)$. Let \mathfrak{k} be the Lie algebra of K . To give the conditions that the Killing spinor equations impose on the fluxes, decompose $L \in \Lambda^2(\mathbb{R}^8) \otimes \mathbb{C}$ and $M \in \Lambda^{4+}(\mathbb{R}^8)$ in irreducible representations of K as

$$L = L^\mathfrak{k} + L^{\text{inv}}, \quad M = M^{\text{inv}} + \tilde{M}, \quad (5)$$

where $L^{\mathfrak{k}}$ is the Lie algebra valued component of L in the decomposition $\Lambda^2(\mathbb{R}^8) = \mathfrak{k} \oplus \mathfrak{k}^{\perp}$, and L^{inv} and M^{inv} are K -invariant two- and four-forms, respectively. M^{inv} decomposes further as $M^{\text{inv}} = m^0 + \hat{M}^{\text{inv}}$, where m^0 has the property that the associated Clifford algebra element satisfies $m^0 \epsilon = g \epsilon$, $g \neq 0$ a spacetime function, for all Killing spinors ϵ . In a particular gauge, the Killing spinor equations imply that g is proportional to Q_- and restrict the spacetime dependence of L^{inv} and M^{inv} . Furthermore, \hat{M} takes values in a representation of K in $\Lambda^{4+}(\mathbb{R}^8)$ with the property that the associated Clifford algebra element satisfies $\hat{M} \epsilon = 0$ for all Killing spinors ϵ . $L^{\mathfrak{k}}$ and \hat{M} are not restricted by the Killing spinor equations.

For compact stability subgroup G , the Killing spinor equations imply the field equations provided the Bianchi identities are satisfied. For the non-compact G , the integrability conditions of the Killing spinor equations and the Bianchi identities imply that all field equations are satisfied provided that $E_{--} = 0$, where E_{--} denotes the $--$ component of the Einstein equations. This in turn gives

$$\begin{aligned} & -(\partial^i + \Omega_j^{ji})(\partial_i V - \partial_v n_I e^I_i) + \frac{1}{4}(dn)_{ij}(dn)^{ij} - \frac{1}{2}\gamma^{IJ}\partial_v^2 \gamma_{IJ} - \frac{1}{4}\partial_v \gamma^{IJ}\partial_v \gamma_{IJ} \\ & - \frac{1}{6}F_{-i_1 \dots i_4} F_{-i_1 \dots i_4} - \frac{1}{4}G_{-i_1 i_2} G_{-i_1 i_2}^* - 2P_- P_-^* = 0, \end{aligned} \quad (6)$$

where γ^{IJ} is the inverse of the metric γ_{IJ} defined in (3). For the special case of fields independent of v , this equation becomes

$$-\square_8 V + \frac{1}{4}(dn)_{ij}(dn)^{ij} - \frac{1}{6}F_{-i_1 \dots i_4} F_{-i_1 \dots i_4} - \frac{1}{4}G_{-i_1 i_2} G_{-i_1 i_2}^* - 2P_- P_-^* = 0, \quad (7)$$

where \square_8 is the Laplacian on the eight-dimensional space Y_8 and dn takes values in \mathfrak{k} . Observe that the spacetime rotation and the fluxes contribute with different relative sign in the field equation as may have been expected.

The backgrounds that we have found can be thought of as vacua of IIB string theory. This particularity applies to compact stability subgroups. The backgrounds $\mathbb{R}^{9-n,1} \times Y_n$ are vacua of IIB compactifications on G_2 for $n = 7$, and on Calabi-Yau manifolds for $n = 6$ and $n = 4$. The backgrounds $AdS_{5-n/2} \times S^{5-n/2} \times Y_n$ can be thought of as either the vacua of the Calabi-Yau or $S^{5-n/2} \times Y_n$ compactifications with fluxes. It is worth pointing out that there are additional vacua associated with the plane waves.

3 $N = 31$ in IIB

The holonomy of the supercovariant connection of type II and $D = 11$ supersymmetric, $N < 32$, backgrounds is a proper subgroup of $SL(32, \mathbb{R})$. In particular for any $N < 32$, there are components of the supercovariant curvature which are not restricted by the gravitino KSE, for M-theory see [17–19] and for IIB see [20]. Furthermore, it was argued in [15] that the Killing spinor bundle \mathcal{K} can be any subbundle of the Spin bundle and the spacetime geometry depends on the trivialization of \mathcal{K} . This is unlike what happens in the case of Riemannian and Lorentzian geometries [14, 21] and heterotic and type I supergravities (provided the parallel spinors are Killing) [22] where there are restrictions both on the number of Killing spinors and the Killing spinor bundle. It is clear from the above arguments that the gravitino KSE allows for the possibility that supersymmetric backgrounds exist for any N . However, the algebraic KSE, Bianchi identities and field equations that supersymmetric backgrounds must satisfy are not included in the holonomy argument. Because of this, the holonomy argument is not conclusive.

To investigate whether there are backgrounds for any N we consider IIB $N = 31$ supersymmetric backgrounds. Backgrounds with 31 supersymmetries have been considered in the context of M-theory [23] and have been called preons. We shall see that the IIB algebraic KSE implies that such backgrounds must be maximally supersymmetric [13]. To our knowledge this is the first example which demonstrates that there are restrictions on the number of supersymmetries of type II backgrounds. To do this, we shall adapt the spinorial method [8] of solving Killing spinor equations to backgrounds that admit near maximal number of supersymmetries. We shall mostly focus on IIB but the analysis extends to $D = 11$ and other lower-dimensional supergravities.

To adapt the spinorial method to backgrounds with near maximal number of supersymmetries, we introduce a “normal” \mathcal{K}^\perp to the Killing spinor bundle \mathcal{K} of a supersymmetric background. The spinors of IIB supergravity are complex positive chirality Weyl spinors, so the Spin bundle is $\mathcal{S}^c = \mathcal{S}_+ \otimes \mathbb{C}$, where \mathcal{S}_+ is the rank sixteen bundle of positive chirality Majorana-Weyl spinors. \mathcal{S}_+^c may also be thought of as an associated bundle of a principal bundle with fibre $SL(32, \mathbb{R})$, the holonomy group of the supercovariant connection, acting with the fundamental representation on \mathbb{R}^{32} . If a background admits N Killing spinors which span the fibre of the Killing spinor bundle \mathcal{K} , then one has the sequence

$$0 \rightarrow \mathcal{K} \rightarrow \mathcal{S}_+^c \rightarrow \mathcal{S}_+^c / \mathcal{K} \rightarrow 0. \quad (8)$$

The inclusion $i : \mathcal{K} \rightarrow \mathcal{S}_+^c$ can be locally described as

$$\epsilon_r = \sum_{i=1}^{32} f^i_r \eta_i, \quad r = 1, \dots, N \quad (9)$$

where η_p , $p = 1, \dots, 16$, is a basis in the space of positive chirality Majorana-Weyl spinors, $\eta_{16+p} = i\eta_p$ and the coefficients f are real spacetime functions. For our notation and spinor conventions see [15]. Any N Killing spinors related by a local $Spin(9, 1)$ transformation give rise to the same spacetime geometry. This is because the Killing spinor equations and the field equations of IIB supergravity are Lorentz invariant. Therefore any bundles of Killing spinors and any choice of sections related by a $Spin(9, 1)$ gauge transformation should be identified.

The normal to the Killing spinor bundle, \mathcal{K}^\perp , is a subbundle of $\mathcal{S}_-^c = \mathcal{S}_- \otimes \mathbb{C}$ defined by the orthogonality condition

$$\mathcal{B}(\nu, \epsilon) = 0, \quad (10)$$

where ν is a section of \mathcal{K}^\perp , $\mathcal{B} = \text{Re } B$ is a non-degenerate inner product, and $B : \mathcal{S}_+ \otimes \mathcal{S}_- \rightarrow \mathbb{R}$ is a $Spin(9, 1)$ -invariant inner product

$$B(\epsilon, \zeta) = -B(\zeta, \epsilon) = \langle B(\epsilon^*), \zeta \rangle, \quad (11)$$

extended bi-linearly on $\mathcal{S}_+^c \otimes \mathcal{S}_-^c$. To write this orthogonality condition in components, introduce a basis in \mathcal{S}_-^c , say $\theta_{i'} = -\Gamma_0 \eta_i$. Then write $\nu = n^{i'} \theta_{i'}$ and the condition (10) can be written as

$$n^{i'} \mathcal{B}_{i'j} f^j_r = 0, \quad (12)$$

where $\mathcal{B}_{i'j} = \mathcal{B}(\theta_{i'}, \eta_j)$.

Let us now consider the IIB $N = 31$ backgrounds. The rank of \mathcal{K}^\perp is one. The spinorial geometry method can be applied as follows. First the $Spin(9, 1)$ gauge symmetry of the IIB KSE can be used to orient the normal spinor along three different directions. This is because there are three kinds of orbits of $Spin(9, 1)$ in the negative chirality Weyl spinors with stability subgroups $Spin(7) \ltimes \mathbb{R}^8$, $SU(4) \ltimes \mathbb{R}^8$ and G_2 , respectively. This can be easily seen using the results of [9]. The three representatives can be chosen as

$$\nu_1 = (n + im)(e_5 + e_{12345}), \quad \nu_2 = (n - \ell + im)e_5 + (n + \ell + im)e_{12345}, \quad (13)$$

$$\nu_3 = n(e_5 + e_{12345}) + im(e_1 + e_{234}), \quad (14)$$

where according to spinorial geometry we have written the spinors as multi-forms. Therefore up to a $Spin(9, 1)$ gauge transformation, \mathcal{K}^\perp can be chosen to lie along one of these three directions. In turn enforcing the orthogonality condition (12), there are three different hyper-planes that the Killing spinors lie in the space of spinors. The expressions for the Killing spinors can be found in [13]. Next, one substitutes the Killing spinors into the IIB algebraic KSE. Then either a direct computation using an oscillator basis in the space of spinors or a straightforward argument based on the expression of Killing spinors in terms of forms

reveals that the flux field strengths P and G vanish, $P = G = 0$. Due to this, the IIB gravitino Killing spinor equation becomes linear over the complex numbers. This means that backgrounds with vanishing P and G fluxes always preserve an even number of supersymmetries. Thus backgrounds with 31 supersymmetries preserve an additional supersymmetry and so they are maximally supersymmetric. Later it was shown that IIA $N = 31$ backgrounds are also maximally supersymmetric in [24]. Thus there are no type II preons.

Next let us consider $N = 31$ backgrounds in eleven dimensions. $D = 11$ supergravity does not have an algebraic KSE and so the analysis presented for such backgrounds in type II theories does not generalize. Nevertheless, the spinorial geometry method can be easily adapted to investigate $N = 31$ supersymmetric backgrounds in eleven dimensions. In particular, one can show that the Killing spinors, after an appropriate choice of the normal spinor up to $Spin(10, 1)$ gauge transformations, take a rather simple form. Next, the holonomy argument indicates that there may be $D = 11$ backgrounds with $N = 31$ supersymmetries. But it turns out that all components of the supercovariant curvature vanish as a consequence of imposing in addition the Bianchi identities and the field equations of the theory. Thus the reduced holonomy of $N = 31$ backgrounds is in fact $\{1\}$ and so these backgrounds admit an additional Killing spinor. Therefore the $N = 31$ backgrounds are locally isometric to maximally supersymmetric ones [25]. Similar results also hold for type I supergravity. Therefore in $D = 10$ and $D = 11$ supergravities there are not supersymmetric backgrounds for all N . This may lead to a simplification in the classification of supersymmetric backgrounds.

Acknowledgements D.R. wishes to thank the organisers of the RTN project ‘Constituents, Fundamental Forces and Symmetries of the Universe’ conference in Napoli for the opportunity to give a talk in a very nice and inspiring atmosphere. Part of the work on which this talk is based was completed while D.R. was a post-doc at King’s College London, for which he would like to acknowledge the PPARC grant PPA/G/O/2002/00475. In addition, he is presently supported by the European EC-RTN project MRTN-CT-2004-005104, MCYT FPA 2004-04582-C02-01 and CIRIT GC 2005SGR-00564. U. G. has a postdoctoral fellowship funded by the Research Foundation K. U. Leuven.

References

- [1] M. Blau, J. Figueroa-O’Farrill, C. Hull, and G. Papadopoulos, J. High Energy Phys. **0201**, 047 (2002) [arXiv:hep-th/0110242].
- [2] P.T. Chrusciel and J. Kowalski-Glikman, Phys. Lett. B **149**, 107 (1984).
- [3] J. Figueroa-O’Farrill and G. Papadopoulos, J. High Energy Phys. **0108**, 036 (2001) [arXiv:hep-th/0105308].
- [4] J. Figueroa-O’Farrill and G. Papadopoulos, J. High Energy Phys. **0303**, 048 (2003): [arXiv:hep-th/0211089].
- [5] O.A.P. MacConamhna, Phys. Rev. D **70**, 105024 (2004) [arXiv:hep-th/0408203].
- [6] J.P. Gauntlett and S. Pakis, J. High Energy Phys. **0304**, 039 (2003) [arXiv:hep-th/0212008].
- [7] J.P. Gauntlett, J.B. Gutowski, and S. Pakis, J. High Energy Phys. **0312**, 049 (2003) [arXiv:hep-th/0311112].
- [8] J. Gillard, U. Gran, and G. Papadopoulos, Class. Quantum Gravity **22**, 1033 (2005) [arXiv:hep-th/0410155].
- [9] U. Gran, J. Gutowski, and G. Papadopoulos, Class. Quantum Gravity **22**, 2453 (2005) [arXiv:hep-th/0501177].
- [10] U. Gran, J. Gutowski, and G. Papadopoulos, Class. Quantum Gravity **23**, 143 (2006) [arXiv:hep-th/0505074].
- [11] U. Gran, P. Lohrmann, and G. Papadopoulos, J. High Energy Phys. **0602**, 063 (2006) [arXiv:hep-th/0510176].
- [12] U. Gran, J. Gutowski, G. Papadopoulos, and D. Roest, Nucl. Phys. B **753**, 118 (2006) [arXiv:hep-th/0604079].
- [13] U. Gran, J. Gutowski, G. Papadopoulos, and D. Roest, arXiv:hep-th/0606049.
- [14] J.M. Figueroa-O’Farrill, Class. Quantum Gravity **17**, 2925 (2000) [arXiv:hep-th/9904124].
- [15] U. Gran, J. Gutowski, G. Papadopoulos, and D. Roest, Class. Quantum Gravity **23**, 1617 (2006) [arXiv:hep-th/0507087].
- [16] J.H. Schwarz, Nucl. Phys. B **226**, 269 (1983).
- [17] C. Hull, arXiv:hep-th/0305039.
- [18] M.J. Duff and J.T. Liu, Nucl. Phys. B **674**, 217 (2003) [arXiv:hep-th/0303140].
- [19] G. Papadopoulos and D. Tsimpis, J. High Energy Phys. **0307**, 018 (2003) [arXiv:hep-th/0306117].
- [20] G. Papadopoulos and D. Tsimpis, Class. Quantum Gravity **20** (2003) L253 [arXiv:hep-th/0307127].
- [21] M. Berger, Bull. Soc. Math. France **83**, 279 (1955).
- [22] U. Gran, P. Lohrmann, and G. Papadopoulos, J. High Energy Phys. **0602**, 063 (2006) [arXiv:hep-th/0510176].

- [23] I.A. Bandos, J.A. de Azcarraga, J.M. Izquierdo, and J. Lukierski, Phys. Rev. Lett. **86**, 4451 (2001) [arXiv:hep-th/0101113];
I.A. Bandos, J.A. de Azcarraga, J.M. Izquierdo, M. Picon, and O. Varela, Phys. Rev. D **69**, 105010 (2004) [arXiv:hep-th/0312266].
- [24] I.A. Bandos, J.A. de Azcarraga, and O. Varela, J. High Energy Phys. **0609**, 009 (2006) [arXiv:hep-th/0607060].
- [25] U. Gran, J. Gutowski, G. Papadopoulos, and D. Roest, arXiv:hep-th/0610331.